Worked Solutions

Edexcel C3 Paper G

1. (*a*) $a + 2d = \ln 18$

$a + 4d = \ln 162$

$2d = \ln 162 - \ln 18$

$d = \frac{1}{2}\ln\frac{162}{18} = \ln\sqrt{\frac{81}{9}} = \ln 3$ **(4)**

(*b*) $a + 2\ln 3 = \ln 18$

$a = \ln 18 - \ln 9 \qquad a = \ln 2$ **(2)**

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2. $\frac{(2x+5)}{\cancel{(2x-3)}(2x+3)} \times \frac{\cancel{(2x-3)}\cancel{(x+1)}}{2\cancel{(x+1)}}$

$\frac{(2x+5)}{2(2x+3)}$ **(6)**

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3. (*a*) $-1 \le \cos x \le 1$ **(2)**

(*b*) fg: $x \mapsto \cos\left(x + \frac{\pi}{2}\right); \quad 0 \le x \le \frac{\pi}{2}$ **(3)**

(*c*) $0 \ge \text{fg}(x) \ge -1$ **(2)**

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4. (*a*) $\cos x = -\frac{24}{25}$

$\sin 2x = 2\sin x \cos x$

$= 2 \times \frac{7}{25} \times \frac{-24}{25} = \frac{-336}{625}$ **(5)**

(*b*) $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta} = \frac{1 - \tan\theta}{1 + \tan\theta}$ **(3)**

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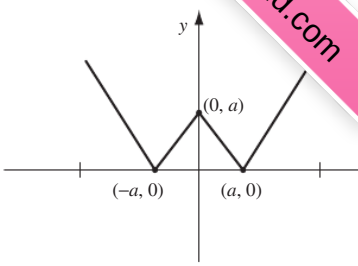
5. (*a*)



(*b*) $\text{f}(2a) = 2(2a) - 2a = 2a$

$\text{f}(-2a) = 2a$

(*c*) $a - x = \frac{1}{2}a \Rightarrow x = \frac{1}{2}a \qquad x = -\frac{1}{2}a$

$2x - 2a = \frac{1}{2}a \quad x = 1\frac{1}{4}a$

$x = -1\frac{1}{4}a$

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6. $\frac{dy}{dx} = e^{2x}(-3\sin 3x + 3\cos 3x) + 2e^{2x}(\cos 3x + \sin 3x)$

$= e^{2x}(-\sin 3x + 5\cos 3x)$

$\frac{d^2y}{dx^2} = 2e^{2x}(-\sin 3x + 5\cos 3x) + e^{2x}(-3\cos 3x - 15$

$= e^{2x}(-17\sin 3x + 7\cos 3x)$

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13 = e^{2x}(-17\sin 3x + 7\cos 3x + 4\sin 3x$

$+13\cos 3x + 13\sin 3x) = 0$

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7. (*a*) $f'(x) = 4 - \frac{1}{x}$

$f'(x) = 0 \Rightarrow x = \frac{1}{4}$

$f(x) = 1 - \ln\frac{1}{4} = 1 + \ln 4 \qquad \left(\frac{1}{4}, 1 + \ln 4\right)$

$f''(x) = +\frac{1}{x^2}$

$f''\left(\frac{1}{4}\right) > 0 \Rightarrow$ min. **(5)**

(*b*) $B(1, 4) \qquad f'(1) = 3$

tangent : $y - 4 = 3(x - 1)$

$y = 3x + 1$

normal $y - 4 = -\frac{1}{3}x + \frac{1}{3}$ **(6)**

(*c*) $P : \left(-\frac{1}{3}, 0\right) \qquad Q : (13, 0)$

$\triangle PBQ = \frac{1}{2}\left(13\frac{1}{3}\right) \times 4 = 26\frac{2}{3}$ **(3)**

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8. (*a*) $R^2 = 5^2 + 12^2$

$\Rightarrow R = 13$

$\tan\alpha = \frac{12}{5}$

$\Rightarrow \alpha = 67.38°$

(*b*) minimum:

$f(x) + 4 = -13 + 4 = -9$

(*c*) $13\sin(x + 67.38) = -13$

$x + 67.38 = 270$

$x = 202.6$

(*d*) $13\sin(x + 67.38) = 6$

$\sin(x + 67.38) = \frac{6}{13}$

$x + 67.38 = 27.48, 152.51, 387.48, \ \ldots$

$x = 85.1, 320.1$
